An analytical method for strength verification of buried steel pipelines at normal fault crossings


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A B S T R A C T

The complex problem of strength verification of a buried steel pipeline crossing the trace of a normal active fault is treated analytically, and a refined methodology for the calculation of the axial and bending pipeline strains is presented. In essence, the proposed methodology extends the analytical methodology originally proposed by Karamitros et al. [1] for the simpler case of strike-slip fault crossings. The modifications introduced to the original methodology are first identified, following a thorough examination of typical results from advanced 3D nonlinear numerical analyses, and consequently expressed via an easy to apply solution algorithm. A set of similar numerical analyses, performed for a wide variety of fault plane inclinations and intersection angles between the pipeline axis and the fault trace, is used to check the accuracy of the analytical predictions. Fairly good agreement is testified for pipeline strains up to 1.50–2.00%. It is further shown that, although the methodology proposed herein applies strictly to the case of right intersection angles, it may be readily extended to oblique intersections, when properly combined with existing analytical solutions for strike-slip fault crossings (e.g. [1]).

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1. Introduction

Buried pipelines are vulnerable to a variety of earthquake-induced hazards, such as permanent ground displacements due to fault rupture or sloping ground failure, or transient ground displacements caused by the passage of seismic waves. Although less frequent, permanent ground displacements pose a higher threat to pipelines, as they may impose large axial and bending strains and they may lead to rupture, either due to tension or due to buckling [2]. This is especially true for step-like deformations resulting from surface faulting, as indicated by a number of case studies of damage to pipeline systems during strong earthquakes (e.g. [3–6]). Taking further into account the critical role of lifeline systems to human life support and energy distribution, as well as the irrecoverable ecological disaster that may result from the leakage of environmentally hazardous materials (e.g. natural gas, fuel or liquid waste), it becomes obvious that the seismic strength verification of buried steel pipelines at active fault crossings is among the top design priorities.

A rigorous solution of the problem should involve an advanced numerical analysis which can account consistently for the nonlinear stress–strain response of the pipeline steel, the longitudinal and transverse soil resistance, typically idealized as a series of elastic–perfectly-plastic distributed (Winkler) springs, as well as second order effects induced by large displacements [7]. Such analyses are definitely possible with currently available commercial computer codes. However, they are rather demanding with regard to computational effort and expertise, so that their use in practice is justified only for the final design of large diameter, thin-walled pipes and large ground displacements. For more common applications, as well as for preliminary pipeline strength verification purposes, it is desirable to use simplified analytical methodologies which will allow reasonably accurate predictions of pipeline stresses and strains, at a fraction of the computational effort required for a rigorous numerical analysis.

Aiming at such a simplified methodology, the mechanisms governing the pipeline response at normal fault crossings are first explored with the aid of results from 3D elastoplastic finite element analyses, for the case of a pipeline with axis perpendicular to the fault trace. We consider this experience of first priority for two reasons. The first reason is to compare with the assumptions of existing analytical methods (e.g. [1,8–10]) and evaluate their capacity to provide a rational solution to this problem. The second, and probably most important reason is to identify the basic features of the pipeline response and focus upon them in order to get the required accuracy while avoiding unnecessary complexities which would make an analytical solution impossible or unfriendly to non-specialists.
Subsequently, the basic principles of the new methodology are outlined and its step-by-step application algorithm is given in detail. Evaluation of the analytical predictions is first obtained from comparison with results from rigorous numerical analyses, for a pipeline crossing the trace of a normal fault at right angle and for various dip angles of the fault plane. Furthermore, the proposed methodology is combined with the original methodology for strike-slip faults [1] to predict the pipeline response for the more general case of oblique crossing between the pipeline axis and the trace of the normal fault.

2. Insight to the mechanisms of pipeline response

To gain insight to the pipeline response at normal fault crossings, a number of representative numerical analyses were conducted with the 3D nonlinear Finite Element method, using the commercial code ANSYS [11]. A typical high-pressure natural gas pipeline was considered, with an external diameter of 0.9144 m (36 in) and a wall thickness of 0.0119 m (0.469 in). A total length of 1000 m was simulated, evenly divided on both sides of the fault trace, discretized into 2000 equal sized pipe elements, each of 0.50 m length. The pipeline steel was of the API5L-X65 type, with a bilinear elasto-plastic stress–strain relationship (Fig. 1) and the properties listed in Table 1. To account for soil–pipeline interaction, each node of the pipeline model was connected to axial, transverse horizontal and transverse vertical soil springs, simulated with elastic–perfectly plastic rod elements (Fig. 2). Their properties, listed in Table 2, were derived according to the ALA-ACSE [7] guidelines, assuming that the pipeline top is buried under 1.30 m of medium-density sand with friction angle $\phi = 36^\circ$ and unit weight $\gamma = 18$ kN/m$^3$. Note that the yield displacements presented in this table were derived by fitting the assumed bilinear force–displacement relationship to the hyperbolic equation proposed by Trautmann and O’Rourke [12], at one half of the ultimate yield force.

Fault displacement was statically applied at the sliding part of the fault, as a permanent displacement of the free end of the corresponding soil-springs (Fig. 2). A dip angle of $\psi = 70^\circ$ relative to the horizontal plane and a total fault displacement of $\Delta f = 2.0D$ were considered, with $D$ being the external diameter of the pipeline. The analysis proceeded incrementally, with a step size of 0.1D. Thus, it was possible to obtain the pipeline response for all intermediate solution stages and explore the evolution of the overall ground–pipeline interaction. The following presentation focuses on four of these intermediate stages, namely for $\Delta f = 0.5D$, 1.0D, 1.5D and 2.0D.

Fig. 3 presents the variation, with distance away from the fault trace, of the pipeline’s axial displacement, axial force and axial strain, as well as of the friction force applied by the surrounding soil. It may be observed that the horizontal component of the applied fault displacement is distributed over a large length of the pipeline, hereafter denoted “unanchored length”, varying from 100D to 350D on both sides of the fault trace. The pipeline elongation within this length results in the development of axial tensile forces, which obtain their maximum value at the intersection point with the fault trace and decrease linearly with the distance away from this point. The linear variation of axial forces is attributed to soil friction, which obtains its limit value along the whole unanchored length. A similar linear variation with the distance away from the fault trace may also be observed for pipeline axial strains, except that two significant peaks develop near the fault trace: one at the foot wall (up-thrown side) of the fault and another, quite smaller, at the hanging wall (down-thrown side). These peaks are attributed to the interaction

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**Table 1**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress ($\sigma_1$)</td>
<td>490 MPa</td>
</tr>
<tr>
<td>Failure stress ($\sigma_2$)</td>
<td>531 MPa</td>
</tr>
<tr>
<td>Yield strain ($\varepsilon_1$)</td>
<td>0.233%</td>
</tr>
<tr>
<td>Failure strain ($\varepsilon_2$)</td>
<td>4.000%</td>
</tr>
<tr>
<td>Elastic Young’s modulus ($E_1$)</td>
<td>210.000 GPa</td>
</tr>
<tr>
<td>Plastic Young’s modulus ($E_2$)</td>
<td>1.088 GPa</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Spring type</th>
<th>Yield force (kN/m)</th>
<th>Yield displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial (friction)</td>
<td>40.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Transverse horizontal</td>
<td>318.6</td>
<td>11.4</td>
</tr>
<tr>
<td>Vertical (upward displacement)</td>
<td>52.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Vertical (downward displacement)</td>
<td>1360.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>
between axial and bending strains, as it will be explained in more detail in subsequent paragraphs.

The variation of vertical displacements, shear forces, bending moments and bending strains, as well as the developing spring forces for upward and downward pipeline displacement relative to the surrounding soil, are presented in Fig. 4. Focusing upon the most intensely deformed, central part of the pipeline, the following three characteristic points are identified:

- Points A and C, on the foot wall and on the hanging wall of the fault, respectively, which are the closest points to the fault trace where the relative vertical pipeline–ground displacement becomes zero.
- Point B which corresponds to the intersection point of the pipeline axis with the fault trace.

The length of pipeline segment ABC, hereafter denoted “curved length”, ranges from 35D to 45D, i.e. it is considerably smaller than the pipeline’s unanchored length. Furthermore, it is not evenly divided on both sides of the fault, with segment BC over the hanging wall being 3–4 times longer than segment AB over the foot wall. This is attributed to the different values of soil resistance for upward and downward pipeline displacement relative to the surrounding soil. More specifically, at the foot wall of the fault (segment AB), soil resistance forces do not reach the corresponding yield value and increase steadily with applied fault movements. On the contrary, at the hanging wall of the fault (segment BC), soil resistance forces reach the yield limit at the
early stages of pipeline deformation, practically over the entire length of this segment. Since upward resistance forces are considerably smaller than their downward counter-parts, this segment accommodates most of the vertical component of the applied fault movement, and is therefore longer than segment AB.

Coming next to shear forces and bending moments developing in the case of small ground displacement ($\Delta f=0.5D$), it may be observed that their distribution along the pipeline axis is typical of an elastic beam behavior. More specifically, the response of segment ABC is, qualitatively at least, similar to that of an elastic beam, with a trianngularly distributed upward load over segment AB (with peak at B), a uniformly distributed downward load over segment BC, as well as a downward applied displacement at point C which is equal to the vertical component of the applied fault displacement.

Nevertheless, this simplified model does not accurately represent the actual pipeline response at larger ground displacements, where the axial tension applied on the pipeline exerts significant effects on bending stiffness and curvature. In this case, plastic strains accumulate at segment AB, where the maximum bending moment occurs. Thus, even though bending strains in this segment significantly increase with increasing fault displacements, the corresponding bending moments decrease. This seemingly contradictory response is attributed to the following reasons:

- As the yielding portion of the pipeline cross-section increases, under the combined action of tension and bending, the pipeline's bending stiffness is reduced [9]. Taking further into account that the pipeline response is displacement and not load controlled, it is realized that this reduction in bending stiffness is eventually accompanied by a bending moment reduction.
- The applied axial tension, combined with the pipeline's step-like deformation mode, has significant geometrical second-order effects on pipeline curvature, resulting also in an overall decrease of the developing bending moments.

Fig. 5 shows the variation with distance away from the fault trace of the axial ($e_a$), as well as the minimum ($e_{min}$) and maximum ($e_{max}$) longitudinal pipeline strains, obtained for the case of large ground displacements ($\Delta f=2.0D$). Observe that the axial strain ($e_a$) does not exhibit a smooth variation, as in the case of small ground displacements (e.g. $\Delta f=0.5D$ in Fig. 3), but develops a distinct local peak at the point of maximum bending strain ($e_a=e_{max}-e_a=e_{a}-e_{min}$) over the foot wall of the fault. This interaction between axial and bending strains at large ground displacements is also a result of the elastoplastic response of the pipeline section. Namely, as maximum strains exceed the yield limit of the pipeline steel, axial strains need to increase locally, so that the integral of associated longitudinal stresses over the cross-section remains equal to the continuously increasing applied axial force.

Note that all above observations are in qualitative agreement with experimental measurements [13] of strain and soil reaction forces developing at HDPE pipelines due to normal fault displacement.

3. Evaluation of existing analytical methodologies

The best known methodology for pipeline strength verification at fault crossings is probably that proposed by Kennedy et al. [9], and consequently adopted by the ASCE [14] guidelines for the seismic design of pipelines, for both strike-slip and normal faults. Kennedy et al. extended the pioneering work of Newmark and Hall [8], by taking into account soil–pipeline interaction in the transverse, as well as in the longitudinal directions. Their methodology is based on the assumption that the whole pipeline section has undergone yielding in the high-curvature zone (ABC in Fig. 4), so that the pipeline's bending stiffness may be ignored. Nevertheless, it has been previously shown that this assumption is not true for the displacement range considered herein. In fact, significant bending moments develop along the curved pipeline length, which persist with increasing fault displacements. It is therefore reasonable to expect that the zero bending stiffness assumption would result in significant over-estimation of the developing bending strains. Note that Karamitros et al. [1], while evaluating the performance of the Kennedy et al. [9] methodology for strike-slip faults, has shown that the analytically predicted strains are accurate only for very large fault displacements (i.e. larger than 1.5$D$), while they may become even one order of magnitude larger than the actual ones for the smaller fault displacements most commonly considered in practice.

In addition to the above limitation, Kennedy et al. [9] consider a plastic hinge at the intersection of the pipeline axis with the fault trace, as shown in Fig. 6, so that relative transverse pipe–soil displacement only occurs at the hanging wall of the fault. Hence, bending strains developing on the part of the pipeline that lies at the foot wall of the fault are effectively ignored. This assumption
comes to bold contradiction with the results of the previously presented numerical analyses, which clearly indicate that the maximum bending strains occur at the foot wall and not at the hanging wall of the fault.

A more refined analytical methodology was introduced by Wang and Yeh [10] for strike-slip fault crossings. They partitioned the pipeline into four (4) distinct segments: two (2) on both sides of the fault trace, within the high-curvature zone, and another two (2) on both sides of this zone. The latter are analyzed as beams-on-elastic-foundation, while the former are assumed to deform as circular arcs, with a radius of curvature calculated from the equations of equilibrium and the demand for continuity between adjacent segments.

This segmentation allows the pipeline’s bending stiffness to be taken into account, thus overcoming the basic limitation of the methodology of Kennedy et al. [9]. Nevertheless, Wang and Yeh overlook the unfavorable contribution of axial forces on the pipeline’s bending stiffness, thus underestimating bending strains. In fact, bending strains are accurately predicted only for very small fault displacements (i.e. up to 0.3D), where no plastic strains develop on the pipeline’s cross-section, while for medium and large fault displacements, they are systematically underpredicted [1].

A number of the afore-mentioned setbacks of the Kennedy et al. [9] and the Wang and Yeh [10] methods have been efficiently eliminated by Karamitros et al. [1] for the case of pipeline crossings with strike-slip faults. More specifically:

- They adopt the pipeline segmentation proposed by Wang and Yeh (1977) with one basic difference: the segments that correspond to the high-curvature zone are not treated as circular arcs, but they are also analyzed as elastic beams, so that the effect of the varying bending stiffness of the pipeline may be accurately taken into account.
- Material nonlinearity is introduced to the solution algorithm by assuming a bilinear stress–strain relationship for the pipeline steel, combined with an iterative equivalent-linear elastic solution scheme.
- Second-order effects, induced by the combination of large displacements and axial tension, are indirectly accounted for, through a simplified equation that combines the bending strains resulting from the elastic beam analysis, with the bending strains calculated assuming that the pipeline has zero bending stiffness.
- The interaction between axial and bending strains is quantified by determining the elasto-plastic distribution of strains and stresses over the pipeline cross-section.

Despite its merits, this methodology is strictly applicable to strike-slip faults, since it assumes a symmetric pipeline deformation about the intersection point of the pipeline axis with the fault trace. As it has been shown in the previous section (e.g. Fig. 4), this assumption is grossly inaccurate for normal faults and may considerably underestimate pipeline strains within the high-curvature zone.

The structural model proposed by Karamitros et al. [1] for strike-slip fault crossings was recently extended to normal fault crossings by Trifonov and Cherniy [15]. More specifically, they removed the symmetry condition about the intersection point, allowing for different types of fault kinematics to be analyzed. In addition, the contribution of transverse displacements has been taken into account for the accurate estimation of the pipeline’s axial elongation. Finally, the two segments in the high-curvature zone are analyzed as beams under combined bending and tension, so that the axial force is directly included to the governing differential equations and the geometrically induced second order effects are consistently taken into account. The above modifications to the original Karamitros et al. [1] methodology have definitely extended its field of application, to strike-slip as well as to normal fault crossings, but at a significant cost of simplicity. This is because the direct approach adopted by Trifonov and Cherniy [15] for simulating second order effects has led to a complex system of equations which can only be solved with involved computationally minimization techniques.

4. Outline of proposed methodology

The proposed methodology aims also to extend the original methodology of Karamitros et al. [1] to normal fault crossings, while maintaining the simplicity of the original solution algorithm. Hence, an analytical solution is initially established for the special case of right (90°) intersection angle between the pipeline axis and the fault trace. It is then shown that the more general case of oblique intersection between the pipeline axis and the fault trace can be decomposed into two essentially uncoupled problems: one of strike-slip fault crossing and the other of normal fault crossing with a 90° intersection angle.

The normal fault considered herein is taken as an inclined plane, with null thickness of the rupture zone, so that the intersection of the pipeline axis with the fault trace on the ground surface is reduced to a single point. The fault displacement is defined in a Cartesian coordinate system, where the X-axis is collinear with the un-deformed longitudinal axis of the pipeline and the Z-axis is vertical. Subsequently, the fault displacement may be analyzed into a horizontal and a vertical component, Δx and Δz, respectively, interrelated through the angle ψ, formed by the horizontal axis and the fault plane (Fig. 7).

Since there is no symmetry around the fault–pipeline intersection point B, the pipeline is partitioned into three segments, as shown in Fig. 7. Partitioning is mainly based on the characteristic points A and C, defined in a previous section as the closest points to the fault trace where vertical pipeline displacements relative to the surrounding soil become equal to zero.

Computation of the combined axial and bending pipeline strains is consequently accomplished in the following six steps:

Step 1: Pipeline segments A′A and CC′ (from −∞ to A and from C to +∞, respectively) are analyzed as beams-on-elastic foundation (Fig. 8), in order to obtain the relation between the shear force, the bending moment and the rotation of points A and C.

Assuming that pipeline deflections along segments A′A and CC′ remain small for the whole range of applied fault displacements, their response is considered elastic and the differential equation for the associated elastic lines is written as

\[ E_1lw'' + kw = 0 \]  

Starting with segment CC′ (Fig. 8b) and imposing \( w = 0 \) for \( x = 0 \) and \( x \to \infty \) yields:

\[ w = Ce^{-\lambda x} \sin \lambda x \]  

with

\[ \lambda = \sqrt{\frac{k}{4E_1l}} \]  

In the above equations, x is the distance from point A along the pipeline axis, w is the pipeline’s vertical displacement, \( E_1 \) is the elastic Young’s modulus of the pipeline steel and k is the vertical soil spring constant. The value of k is different for upward and downward pipeline displacement (e.g. [7]). Still, to reduce the number of variables, it is possible to use an average value for the entire segment CC′, regardless of the direction of pipeline
displacement, with only minor effect on the overall pipeline response.

Differentiation of Eq. (2) yields the following relations between the shear force $V_C = -E_1 I w_C$ and the bending moment $M_C = -E_1 I w_C$ at point C. The convention for positive internal forces ($M$ and $V$) and displacements ($w$ and $\phi$) used in these relations is shown in Fig. 9. Note that in Fig. 8, internal forces and displacements are shown with their actual and not the conventional directions, so that the loading and deformation patterns of the pipeline may be more clearly visualized.

$$M_C = (2\bar{E} I) \phi_C$$

$$V_C = -\lambda M_C$$

Since $k$ is practically the same for both segments, a similar procedure may be followed for segment $A'A$ (Fig. 8a). More specifically, the condition of $w=0$ is imposed for $x=0$ and $x \to -\infty$, yielding:

$$w = Ce^{\lambda x} \sin \lambda x$$

Thus, the following relations may be derived between the shear force $V_A = -E_1 I w_A$, the bending moment $M_A = -E_1 I w_A$ and the rotation $\phi_A = w_A$ at point C:

$$M_A = -(2\bar{E} I) \phi_A$$

$$V_A = \lambda M_A$$

**Step 2**: The relations obtained in Step 1 are applied as boundary conditions to central segment ABC in order to compute the associated maximum bending moment.

Pipeline segment ABC (Fig. 10) is analyzed as an elastic beam of total length $L$ and bending stiffness $EI$. The length $L$ is equal to the sum of lengths $L_{AB}$ and $L_{BC}$ of pipeline segments AB and BC, lying on the hanging and foot walls of the fault, respectively. Points A and C are supported by rotational springs, whose constant is calculated from Eqs. (3) and (6) as $C_r = 2\bar{E} I L$.

Consistent with the results of the numerical analyses in Section 2, a vertical displacement is applied at point C, equal to the vertical component of the fault displacement $\Delta z$. Furthermore, a uniformly distributed load $q_{BC}$ is applied to pipeline segment BC, equal to the limit value of the resistance force, for upward pipeline displacement relative to the surrounding soil. As explained in Section 2, soil reaction forces developing at segment AB do not exceed the corresponding yield limit, for the entire range of examined fault displacements. Thus, the actual applied load along this segment is proportional to the downward displacement of the pipeline, being equal to zero at point A and...
obtaining its maximum value at point B. Nevertheless, for the sake of simplicity, a uniformly distributed load $q_{AB}$ will be also applied to segment AB (Fig. 10), equal to one half of unit soil resistance forces developing at point B, i.e.:

$$q_{AB} = \frac{k_{down} \Delta z_B}{2}$$

where $k_{down}$ is the stiffness of the respective soil springs. Computation of the vertical displacement $\Delta z_B$ follows an approximate procedure which radically simplifies the resulting equations. Namely, segments AB and BC are simulated as circular arcs subjected to an axial tensile force $F_a$, as well as to the uniformly distributed loads $q_{AB}$ and $q_{BC}$ (Fig. 11a). As described in Section 2, the axial force developing at the pipeline obtains its maximum value at the intersection point with the fault trace, and decreases linearly with the distance from this point. However, this decrease is due to the applied soil friction and occurs within the “unanchored length” of the pipeline, which is one order of magnitude larger than the “curved length”. Therefore, in this Step, the axial force is assumed to remain constant along the lengths AB and BC. Taking the above into account, the respective radii of curvature $R_{AB}$ and $R_{BC}$ may be readily calculated considering equilibrium of an infinitesimal arc of these segments (Fig. 11b), as

$$R_{AB} = \frac{F_a}{q_{AB}}$$

$$R_{BC} = \frac{F_a}{q_{BC}}$$

Note that the above assumption is equivalent with neglecting the bending stiffness of segment ABC, as if it behaved as a cable. This is similar to what Kennedy et al. [9] have assumed in their pioneering work, resulting in the prediction of excessive pipeline strains. Nevertheless, this danger does not exist here as the zero bending stiffness assumption is only considered for the simplified calculation of $\Delta z_B$, while it is reinstated in the remaining computations.

For the simplified mode of deformation of Fig. 11, $\Delta z_B$ may be computed in advance, in terms of known problem variables, as

$$\Delta z_B = \frac{-q_{BC} + \sqrt{q_{BC}^2 + 2k_{down} \Delta z}}{k_{down}}$$

Given the displacement $\Delta z_B$, segment ABC may be analyzed according to the elastic-beam theory. More specifically, bending moments $M_A$ and $M_C$, developing at points A and C, respectively,
may be computed from Eqs. (11a) and (11b). As in Step 1, this mathematical derivation follows the convention for positive internal forces and displacements presented in Fig. 9. Therefore, $M_A$ is expected to be negative, while $M_C$ is expected to be positive, so as to comply with the deformation pattern illustrated in Fig. 10.

\[ M_A = \frac{4EI}{L} \phi_A + 2\frac{EI}{L} \phi_C - 6\frac{EI}{L^2} \Delta z \]

\[ + \frac{q_{AB}l_{AB}^2}{12} (6 - 8\frac{L_{AB}}{L} + 3\frac{l_{AB}^2}{L^2}) - \frac{q_{BC}l_{BC}^2}{12L} (4 - 3\frac{L_{BC}}{L}) \]

\[ M_C = -2\frac{EI}{L} \phi_A - 4\frac{EI}{L} \phi_C + 6\frac{EI}{L^2} \Delta z + \frac{q_{AB}l_{AB}^2}{12} (4 - 3\frac{L_{AB}}{L}) \]

\[ - \frac{q_{BC}l_{BC}^2}{12} (6 - 8\frac{L_{BC}}{L} + 3\frac{l_{BC}^2}{L^2}) \]

(11a)

(11b)

Combining the above with Eqs. (3) and (6), the following relations for the computation of bending moments $M_A$ and $M_C$:

\[ M_A = \frac{(2 + (C/L/2EI))M_0^b = 0 + M_0^b = 0}{4 + (6EI/C/L) + (C/L/2EI)} \]

\[ M_C = \frac{M_0^c = 0 + (2 + (C/L/2EI))M_0^c = 0}{4 + (6EI/C/L) + (C/L/2EI)} \]

where

\[ \frac{M_0^b = 0 = -6\frac{EI}{L^2} \Delta z + \frac{q_{AB}l_{AB}^2}{12} (6 - 8\frac{L_{AB}}{L} + 3\frac{l_{AB}^2}{L^2}) - \frac{q_{BC}l_{BC}^2}{12L} (4 - 3\frac{L_{BC}}{L})}{4 - 3\frac{L_{AB}}{L} + 3\frac{l_{BC}^2}{L^2}} \]

\[ \frac{M_0^c = 0 = 6\frac{EI}{L^2} \Delta z + \frac{q_{AB}l_{AB}^2}{12} (4 - 3\frac{L_{AB}}{L}) - \frac{q_{BC}l_{BC}^2}{12L} (6 - 8\frac{L_{BC}}{L} + 3\frac{l_{BC}^2}{L^2})}{4 - 3\frac{L_{AB}}{L} + 3\frac{l_{BC}^2}{L^2}} \]

(13a)

(13b)

Given the bending moments at points A and C, the corresponding shear forces may be computed by considering moment equilibrium about points C and A, respectively:

\[ V_A = \frac{1}{L} \left[ -M_A + M_C - q_{AB}L_{AB} \left( 1 - \frac{L_{AB}}{2} \right) + \frac{q_{AB}l_{AB}^2}{2} \right] \]

\[ V_C = \frac{1}{L} \left[ -M_A + M_C + \frac{q_{AB}l_{AB}^2}{2} - q_{BC}L_{BC} \left( 1 - \frac{L_{BC}}{2} \right) \right] \]

(14a)

(14b)

Note that, in the above equations, the curved lengths $L_{AB}$ and $L_{BC}$ are not a priori known, and must be estimated iteratively, taking into account the boundary conditions for pipeline segment ABC that were obtained from the analysis of segments A and C (Eqs. (4) and (7)). More specifically, initial values of $L_{AB} = 5$–$10D$ and $L_{BC} = 20$–$25D$ are first assumed, reaction forces at points A and C are consequently computed using Eqs. (12) and (14), and the curved lengths $L_{AB}$ and $L_{BC}$ are updated as follows:

\[ L_{AB}' = q_{BC}L_{BC} + V_A - \frac{1}{2}M_A + (1 - \frac{1}{2})l_{AB} \]

\[ L_{BC}' = q_{AB}L_{AB} + V_A + \frac{1}{2}M_C + (1 - \frac{1}{2})l_{AB} \]

(15a)

(15b)

To ensure convergence of the above iterative procedure, the coefficient $\alpha$ in Eqs. (15a) and (15b) should be chosen between 0.20 and 0.50. Note that lengths $L_{AB}$ and $L_{BC}$ calculated from the above equations are appropriate for the estimation of the maximum bending moment and strain, as discussed in Step 4, but they are grossly approximate for the computation of the actual curved length of the pipeline, which also depends on geometrically induced second order effects.

As explained in Section 2, the maximum bending moment $M_{max}$ develops in segment AB, on the foot wall (up-thrown side) of the fault. Given the reaction forces at points A and C, this bending moment may be computed as

\[ M_{max} = M_A + V_Ax_{max} + q_{AB} \frac{x_{max}^2}{2} \]

(16a)

where

\[ x_{max} = -\frac{V_A}{q_{AB}} \]

(16b)

Step 3: Axial strains developing along the pipeline are integrated to compute the pipeline elongation, which is consequently identified as the $\Delta x$ component of the applied fault displacement, in order to derive the maximum axial force.

The maximum axial force, which develops at the intersection of the pipeline with the fault trace, may be calculated, based on the demand for compatibility between the geometrically required and the available pipeline elongation. The geometrically required elongation $\Delta \text{req}$ is defined as the elongation imposed to the pipeline due to the fault displacement. In cases where the normal fault plane is inclined ($\psi < 90^\circ$), and the pipeline poses adequate bending resistance, the vertical component $\Delta z$ of the fault displacement may be considered to have a negligible effect on pipeline elongation compared to the horizontal component $\Delta x$, so that

\[ \Delta \text{req} \approx \Delta x \]

(17)

On the other hand, the available elongation $\Delta \text{av}$ may be defined as the integral of axial strains developing along the pipeline’s unanchored length $L_{anch}$, on each side of the fault trace, where slippage occurs between the pipeline and the surrounding soil, i.e.:

\[ \Delta \text{av} = 2 \int_0^{L_{anch}} \varepsilon(L) dL \]

(18)

where $L$ denotes the distance from the fault trace.

Taking into account that the soil friction forces which develop along the pipeline’s unanchored length $L_{anch}$ are equal to the limit value $t_0$, the axial pipeline stresses and the unanchored pipeline length may be computed as

\[ \sigma(L) = \sigma_a - \frac{t_0}{A_t} L \]

(19)

\[ L_{anch} = \frac{F_a}{t_0} = \frac{\sigma_a A_t}{t_0} \]

(20)

where $F_a$ and $\sigma_a$ are the axial force and stress developing at the intersection of the pipeline axis with the fault trace, while $A_t$ is the area of the pipeline cross-section.

Assuming further a bilinear stress-strain relationship for the pipeline steel (Fig. 1), Eq. (19) may be used to derive the distribution of axial strains along the pipeline’s unanchored length and consequently calculate the available elongation $\Delta \text{av}$. Namely, in case that the maximum tensile stress $\sigma_a$ is smaller than the pipeline’s yield stress $\sigma_y$, pipeline strains remain elastic (Fig. 12a) and would be expressed as

\[ \varepsilon(L) = \frac{\sigma(L)}{E_t} \]

(21)

The available elongation may be analytically computed as

\[ \Delta \text{av} = 2 \int_0^{L_{anch}} \frac{\sigma(L) - (t_0/A_t)L}{E_t} dL = \frac{\sigma_a A_t}{E_t t_0} \]

(22)

while for $\Delta \text{req} = \Delta x$, the maximum tensile stress becomes

\[ \sigma_a = \frac{E_t t_0 \Delta x}{A_t} \]

(23)
If the required elongation is larger than that corresponding to 
\( \sigma_a = \sigma_1 \), i.e. when

\[
\Delta x > \Delta x_{\text{eq},\ell} \quad \Rightarrow \quad \Delta x = \frac{\sigma^2 A_b}{E_1 t_u} \quad (24)
\]

plastic strains develop on the pipeline (Fig. 12b) and the corresponding available elongation becomes

\[
\Delta L_{\text{av}} = 2 \int_0^{l_1} \left( \varepsilon_1 + \frac{\sigma_1(E_1 - E_2)}{E_2} \right) \, dL + \int_{l_1}^{l_{\text{anch}}} \frac{\sigma_1(L)}{E_1} \, dL \quad (25a)
\]

where

\[
L_1 = (\sigma_0 - \sigma_1) A_b \quad (25b)
\]

The maximum developing tensile stress consequently becomes

\[
\sigma_a = \sigma_1 (E_1 - E_2) \left( \frac{\sigma_1}{E_1} \right) + \frac{\sigma_1^2 (E_1^2 - E_2^2)}{E_2} + E_2 E_1 \Delta x (t_u / A_b) \quad (26)
\]

In either case (Fig. 12a or b), the corresponding axial force is equal to

\[
F_a = \sigma_0 A_b \quad (27)
\]

Step 4: Maximum bending strains are computed, taking into account second-order effects induced by the tensile force \( F_a \).

Bending strains developing at the point of the maximum bending moment \( M_{\text{max}} \) may be calculated according to the elastic beam theory as

\[
\varepsilon_b = \frac{M_{\text{max}} D}{2EI} \quad (28)
\]

where \( D \) is the pipeline diameter. According to Eq. (28), \( \varepsilon_b \) increases with increasing applied fault displacements. This is not only due to the anticipated decrease of the equivalent Young’s modulus \( E \) when the pipeline steel undergoes yielding, but also due to the increase of the maximum bending moment \( M_{\text{max}} \) calculated according to Eq. (16). However, the numerical analyses presented in Section 2 indicate that the increase of bending moments is limited by geometrical second-order effects, arising from the combination of the vertical component of the fault displacement and the developing axial tension that was calculated in Step 3. As a result, Eq. (28) is expected to overpredict bending strains for large applied fault displacements.

In order to take into account the afore-mentioned second-order effects, an upper limit to these strains may be calculated by assuming that the whole pipeline cross-section has undergone yielding, so that the pipeline’s bending stiffness becomes negligible. In this case, the pipeline would essentially behave like a cable, and its deformation mode would consist of two circular arcs, as described in Fig. 11. Thus, the resulting bending strains would become

\[
\varepsilon_b^{\text{II}} = D / (2R_{\text{AB}}) = \frac{q_{\text{AB}} D}{2F_a} \quad (29)
\]

According to Eq. (29), bending strains \( \varepsilon_b^{\text{II}} \) are inversely proportional to the developing axial force \( F_a \), thus tending to become infinite for small fault displacements.

Summarizing the above, Eq. (28) takes into account the pipeline’s bending stiffness, neglecting second-order effects, and consequently it is only valid for small fault displacements, while it overestimates bending strains for larger applied fault displacements. On the contrary, Eq. (29) accounts for second-order effects, without taking the pipeline’s bending stiffness into consideration, and consequently it overestimates bending strains for small fault displacements while it provides accurate results for large fault displacements, when the whole pipeline cross-section has undergone yielding. Thus, to combine the two, Eq. (30) is introduced which asymptotically tends to \( \varepsilon_b^{\text{II}} \) when fault displacements tend to zero while it tends to \( \varepsilon_b^{\text{I}} \) when fault displacements become very large:

\[
\frac{1}{\varepsilon_b} = \frac{1}{\varepsilon_b^{\text{I}}} + \frac{1}{\varepsilon_b^{\text{II}}} \quad (30)
\]

Step 5: The maximum pipeline strain is computed from stress equilibrium over the pipeline cross section, taking also into account the bilinear elasto-plastic stress–strain relationship for the pipeline steel.

The numerical results of Section 2 indicate that axial strains may increase locally at the point of maximum bending strains, as the pipeline section starts yielding under the combined action of tension and bending. To quantify this effect, it is first necessary to
define the elasto-plastic distribution of longitudinal (axial and bending) stresses over the pipeline’s cross-section. For this purpose, the bilinear stress–strain relationship for the pipeline steel (Fig. 1) will be combined with the beam theory assumption of plane cross-sections displayed in Fig. 13. Similar to Step 4, computations refer to the point of the maximum bending moment $M_{\text{max}}$.

Following the above assumptions, the longitudinal strain and stress distribution over the pipeline cross-section are expressed as

$$
\varepsilon = \varepsilon_a + \varepsilon_b \cos \theta
$$

and are calculated as

$$
\sigma = \begin{cases} 
\sigma_1 + E_t (\varepsilon - \varepsilon_1) & 0 \leq \phi \leq \phi_1 \\
E_t c & \phi_1 \leq \phi \leq \pi - \phi_2 \\
-\sigma_1 + E_t (\varepsilon + \varepsilon_1) & \pi - \phi_2 \leq \theta \leq \pi 
\end{cases}
$$

where $\theta$ is the polar angle of the cross-section, $\sigma_1$ and $\varepsilon_1$ are the yield stress and strain limits of the pipeline steel, while angles $\phi_{1,2}$ define the portion of the cross-section which is under yield and are calculated as

$$
\phi_{1,2} = \begin{cases} 
\pi & \frac{\varepsilon_1 + \varepsilon_b}{\varepsilon_a} > 1 \\
\arccos \left( \frac{\varepsilon_1 + \varepsilon_b}{\varepsilon_a} \right) & 1 \leq \frac{\varepsilon_1 + \varepsilon_b}{\varepsilon_a} \leq 1 \\
0 & \frac{\varepsilon_1 + \varepsilon_b}{\varepsilon_a} < 1
\end{cases}
$$

The corresponding axial force is then calculated as

$$
F = 2 \int_0^\pi \sigma_R d\theta = 2R_m \left[ E_t \pi \varepsilon_a - (E_1 - E_2) (\sigma_1 - \sin \phi_2) \varepsilon_a - (E_1 - E_2) \sigma_1 \sin \phi_2 \right] + (E_1 - E_2) (\sigma_1 - \sin \phi_2) \varepsilon_a - (E_1 - E_2) \sigma_1 \sin \phi_2 \varepsilon_a
$$

where $R_m = (D - t)/2$ is the average pipeline radius.

Equalizing the independently computed axial forces from Eqs. (33) and (27) allows derivation of the pipeline’s axial strain $\varepsilon_a$. The solution is obtained iteratively, using the Newton–Raphson method. Namely, using an initial value of $\varepsilon_a = 0$, the axial strain for each successive iteration can be calculated as

$$
\varepsilon_a^{k+1} = \varepsilon_a^k - \frac{F(\varepsilon_a^k) - F_2}{dF/d\varepsilon_a |_{\varepsilon_a^k}}
$$

where

$$
dF/d\varepsilon_a = 2R_m \left[ E_t \pi - (E_1 - E_2) (\sigma_1 + \sin \phi_2) \varepsilon_a - (E_1 - E_2) (\sigma_1 - \sin \phi_2) \varepsilon_a - (E_1 - E_2)(\sigma_1 + \sin \phi_2) \varepsilon_a - (E_1 - E_2)(\sigma_1 - \sin \phi_2) \varepsilon_a \right]
$$

Having thus calculated both the axial and bending strains, the maximum and minimum longitudinal strains can be computed from Eq. (31a), for $\theta = 0$ and $\pi$, respectively.

Step 6: Given the stress and strain distribution over the pipeline cross-section, an updated secant Young’s modulus is computed in order to take into account the elasto-plastic response of the pipeline steel.

In the above-described procedure, pipeline strains are a function of the maximum bending moment within segment ABC, which has been computed on the basis of elastic beam theory, i.e. without taking explicitly into account the elasto-plastic behavior of the pipeline steel. To correct this, the stresses and strains computed above may be used in connection with the bilinear relationship of Fig. 1 in order to derive an equivalent secant Young’s modulus for the pipeline steel, and repeat steps 2–6 until convergence is achieved. The secant modulus that is used in each iteration is calculated as

$$
E_{\text{sec}} = \frac{M_{\text{sec}} D}{2t} \left( \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right)
$$

where bending moment $M_{\text{sec}}$ is computed from the already defined stress distribution over the pipeline cross-section, as

$$
M_{\text{sec}} = 2 \int_0^\pi \sigma_R d\theta d\theta = M_{\text{max}}
$$

5. Validation of the proposed methodology

To be able to establish an analytical solution algorithm which can be readily solved without involved numerical techniques, it was necessary to adopt a number of simplifying assumptions, regarding the soil–pipeline interaction. Although the selection of these assumptions was not intuitive, but it was guided by the inspection of typical numerical predictions in Section 2, the overall accuracy of the proposed methodology was further checked against the results of rigorous 3D non-linear Finite Element analyses. The mesh discretization, the pipeline geometry and steel properties, as well as the properties of the surrounding
soil were kept the same as in Section 2, while three different fault plane angles were considered, $\psi = 55^\circ, 70^\circ$ and $85^\circ$. In each case, a total fault displacement of $D_f = 2.0D$ was applied incrementally, with a step size of $0.1D$.

The numerical results are compared to the corresponding analytical predictions in Fig. 14. The comparison is made in terms of the variation, with the normalized fault displacement $D_f / D$, of the peak values of axial strain $e_a$, bending strain amplitude $e_b$ and total strains $e_{\text{max}} = e_a + e_b$ and $e_{\text{min}} = e_a - e_b$. A fairly good agreement may be testified, for all components of pipeline strain, for the first two cases of fault plane inclinations ($\psi = 55^\circ$ and $70^\circ$). For the third case of a nearly vertical fault plane ($\psi = 85^\circ$), which is rather unusual in practice, the agreement is equally good for fault displacements up to $1.5D$ (maximum total strain $e_{\text{max}} \approx 1\%$). For larger fault displacements, the proposed methodology underestimates both the axial strain $e_a$ and the bending strain amplitude $e_b$. This is mainly a result of neglecting the effect of the vertical component $\Delta z$ of the fault displacement on the overall pipeline elongation. This simplifying assumption is realistic for small pipeline strains and inclined fault planes, but becomes grossly unrealistic as yielding extends over a major portion of the pipeline cross section, creating essentially a plastic hinge at the intersection with the fault trace, while the fault plane tends also to become vertical.

6. Application to oblique fault crossings

The methodology proposed in the previous sections has been developed for normal fault crossings, where the pipeline axis is perpendicular to the fault trace. However, it is quite common in practice to encounter oblique crossings, where the pipeline axis crosses the fault trace at an angle $\beta \neq 90^\circ$ (Fig. 15), and the pipeline displacements develop in the vertical but also in the horizontal plane through the pipeline axis. In that case, the Cartesian coordinate system which was used in the previous solution scheme has to be redefined. Namely, the $X$-axis remains collinear with the un-deformed longitudinal axis of the pipeline, the $Z$-axis is vertical, while a $Y$-axis is added in the horizontal plane. The fault displacement $\Delta f$ may be consequently analyzed into three components, interrelated through angles $\psi$ and $\beta$, defined in Fig. 15:

$$\Delta x = \Delta f \cos \psi \sin \beta$$  \hspace{1cm} (37a)
\[ \Delta y = \Delta f \cos \psi \cos \beta \quad (37b) \]
\[ \Delta z = \Delta f \sin \psi \quad (37c) \]

This general 3D case may be approximately decomposed into two simpler 2D cases with known analytical solutions: a strike-slip fault crossing with \( \psi = 0^\circ \) and \( \beta < 90^\circ \), which can be analyzed with the methodology of Karamitros et al. [1], and a normal fault crossing with \( \beta = 90^\circ \) and \( \psi > 0^\circ \), which will be analyzed with the methodology proposed herein.

In order to check the accuracy of this approach, 3D nonlinear Finite Element analyses were performed for three different cases of oblique fault crossing, with \( \beta = \psi = 30^\circ \), \( 45^\circ \) and \( 60^\circ \). In the first case, the transverse horizontal component of the applied fault displacement clearly prevails over the vertical component, while in the last case the roles are reversed. The numerical algorithm, as well as the pipeline characteristics and the soil properties, were the same as the ones presented in Sections 2 and 5 before.

The corresponding numerical results are summarized in Fig. 16, with the same format as in Fig. 14. The only difference is that now the numerical predictions are compared to two sets of analytical predictions:

- one for strike-slip fault crossing with \( \Delta x = \Delta f \cos \psi \sin \beta \) and \( \Delta y = \Delta f \cos \psi \cos \beta \), obtained with the methodology of Karamitros et al. [1], and

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**Fig. 15.** Definition of axes \( x \), \( y \) and \( z \) and fault displacements \( \Delta x \), \( \Delta y \) and \( \Delta z \), for oblique fault crossings.

**Fig. 16.** Evaluation of analytical predictions for oblique fault crossing against results from 3D nonlinear Finite Element analyses.
• another for normal fault crossing with \( \Delta x = A_1 \cos \psi \sin \beta \) and \( \Delta z = A_1 \sin \psi \), obtained with the methodology presented in this paper.

It may be observed that, regardless of the prevailing pattern of fault displacements (strike-slip or normal fault), the numerical predictions are in fairly good agreement with the maximum, in absolute values, of the strains calculated using the strike-slip fault crossing and the normal fault crossing analytical methodologies. Thus, in the first case, where the transverse horizontal component of the applied fault displacement is clearly predominant, the numerical results are fairly well reproduced by the methodology of Karamitros et al. [1]. On the other hand, in the second and third cases, where the vertical component increases relative to the horizontal, the numerical results gradually approach the analytical predictions with the methodology proposed herein.

The reason which justifies the proposed decoupling of permanent fault displacements, in this highly nonlinear problem of soil–structure interaction, is that maximum bending moments and horizontal, the numerical results gradually approach the analytical predictions with the methodology proposed herein.

7. Conclusions

The analytical methodology for the stress–strain analysis of buried steel pipelines crossing active strike-slip faults, proposed by Karamitros et al. [1], is herein extended to the case of normal faults. Comparison of the analytical predictions with the results of benchmark numerical analyses, performed over a wide range of fault displacements, fault plane inclinations and intersection angles, showed fairly good overall agreement, with minor deviations which did not generally exceed about 10–20%.

It should be noted that, in its present form, the proposed methodology applies under the following conditions and limitations:

(a) The pipeline crosses normal or oblique faults that result in elongation of the pipeline, with tension and bending being the prevailing modes of deformation.

(b) The pipeline axis is practically straight, while slippage of the pipeline relatively to the surrounding soil is not restrained along the pipeline’s unanchored length (e.g. due to bends, anchor plates, etc.).

(c) The effects of local buckling and section deformation [16] are not addressed, and consequently the proposed methodology should not be extended beyond the strain limits explicitly defined by design codes (e.g. [7,17]), in order to mitigate such phenomena in practice.

(d) Large deviations between analytical and rigorous numerical predictions were witnessed for nearly vertical fault planes and relatively large fault displacements (e.g. above 1.25D) where a large part of the pipeline section has undergone yield.

Although the practical significance of the above disclaimers cannot be undervalued, it should be acknowledged that the proposed methodology provides accurate analytical predictions for a wide range of pipeline–fault-soil conditions encountered in practice, while it remains relatively simple and stable and can be easily programmed for quick application. Still, for the convenience of interested users, the software for spread sheet application of the proposed methodology is available at the web site of the second author (www.georgebouckovalas.com).

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